This strategy notebook is designed to be a reference for teachers when they are teaching the strategy standards in whole group. **STUDENTS DO NOT HAVE TO MASTER ALL THESE STRATEGIES. THESE ARE JUST EXAMPLES OF WAYS FOR STUDENTS TO BUILD NUMBER SENSE. THE STRATEGIES THAT SAY “IMPORTANT” ARE THE ONES THAT STUDENTS NEED TO UNDERSTAND.**
Repeated Addition

This is a low level strategy used by students who do not yet break numbers into tens and ones.

\[
\begin{array}{c}
63 \times 5 \\
63 \\
+ 63 \\
\underline{126} \\
\hline
63 \\
63 \\
\hline
189 \\
63 \\
\hline
252 \\
63 \\
\hline
315
\end{array}
\text{ or }
\begin{array}{c}
63 \\
126 \\
63 \\
\underline{189} \\
\hline
63 \\
126 \\
\underline{189} \\
\hline
315
\end{array}
\]

Partitioning Strategies

Students break numbers up in a variety of ways reflecting their understanding of base-ten concepts.

By Decades:

\[
\begin{array}{c}
63 \times 5 \\
60 \times 5 = 300 \\
3 \times 5 = 15 \\
\hline
315
\end{array}
\]

By Tens and Ones:

\[
\begin{array}{c}
63 \times 5 \\
10 \times 5 = 50 \\
10 \times 5 = 50 \\
10 \times 5 = 50 \\
10 \times 5 = 50 \\
10 \times 5 = 50 \\
10 \times 5 = 50 \\
3 \times 5 = 15 \\
\hline
300 \\
\hline
315
\end{array}
\]
Making Landmark and Friendly Numbers
Strategy

Sometimes a multiplication problem can be made easier by changing one of the factors to a “friendly" or “landmark number”.

- Friendly numbers are numbers that end in 0. They are called friendly because once the rule for multiplying 0 is understood, that understanding can be extended to larger numbers that end in 0.
- Landmark numbers are similar to friendly numbers. Some examples are 25, 50, 75, 100.

A common error students make is that they forget to adjust the number of groups. Look at this problem: 9 x 25. If the 9 is changed to 10 (friendly number), then the product of 250 would need to be adjusted not just by 1 but by one group of 25.

<table>
<thead>
<tr>
<th>25</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>x 9 + 1 x 10</td>
<td>Common Error</td>
</tr>
<tr>
<td>250 - 1 = 249</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>25</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>x 9 + 1 x 10</td>
<td>Correct Way</td>
</tr>
<tr>
<td>250 - 25 = 225</td>
<td></td>
</tr>
</tbody>
</table>

Here are some problems you can use that support “Making Landmark or Friendly Numbers.”

<table>
<thead>
<tr>
<th>5 x 19</th>
<th>4 x 49</th>
<th>4 x 249</th>
<th>8 x 199</th>
<th>36 x 19</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 x 25</td>
<td>53 x 48</td>
<td>36 x 20</td>
<td>5 x 359</td>
<td>12 x 149</td>
</tr>
</tbody>
</table>
"Partial Products" Strategy (Important)

This strategy is based on the distributive property. When students break apart factors into addends (through expanded form and distributive property), then they can use the smaller problems to solve more difficult ones. As students invent “partial product” strategies, they can break one or both factors apart. While both factors can be represented with expanded notation, it is sometimes more efficient to keep one number whole.

Example of "Partial Products" with the problem 12 x 15:

\[
\begin{align*}
12 \times 15 & \quad \text{In this example, the 15 is} \\
12 \times 15 \ (10 + 5) & \quad \text{changed to (10 + 5) while} \\
12 \times 10 = 120 & \quad \text{the 12 is kept whole.} \\
12 \times 5 = 60 & \\
120 + 60 = 180 & \\
\end{align*}
\]

\[
\begin{align*}
12 \times 15 & \quad \text{This time the 12 is broken up} \\
(4 + 4 + 4) \times 15 & \quad \text{and the 15 is kept whole. The} \\
4 \times 15 = 60 & \quad 12 \text{ could have been broken any} \\
4 \times 15 = 60 & \quad \text{way.} \\
4 \times 15 = 60 & \\
60 + 60 + 60 = 180 & \\
\end{align*}
\]

\[
\begin{align*}
12 \times 15 & \quad \text{This time both factors were broken apart. This is more common when the} \\
(10 + 2) \times (10 + 5) & \quad \text{numbers get bigger. Sometimes it is} \\
10 \times 10 = 100 & \quad \text{difficult to do this mentally and easier} \\
10 \times 5 = 50 & \quad \text{with paper and pencil.} \\
2 \times 10 = 20 & \\
2 \times 5 = 10 & \\
100 + 50 + 20 + 10 = 180 & \\
\end{align*}
\]

\[
\begin{align*}
\frac{15}{12} \\
(5 \times 2) = 10 \\
(2 \times 10) = 20 & \quad \text{This time the problem is} \\
(10 \times 5) = 50 & \quad \text{written vertically.} \\
(10 \times 10) = 100 & \\
180 & \\
\end{align*}
\]
Here are some problems you can use that support “Partial Products.”

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>13 x 15</td>
<td>14 x 16</td>
<td>15 x 33</td>
<td>4 x 532</td>
<td>8 x 256</td>
</tr>
<tr>
<td>15 x 11</td>
<td>25 x 14</td>
<td>35 x 24</td>
<td>8 x 112</td>
<td>6 x 325</td>
</tr>
</tbody>
</table>

An area model can be used to model the student’s strategy of partial products. CCGPS states that the student should “illustrate and explain the calculation by using equations, rectangular arrays, and/or area models”.

The array model allows students to prove their reasoning. This array shows how to break apart 12 x 15 into (10 + 2) x (10 + 5).
"Doubling and Halving" Strategy

Consider all the different arrays we can make with an area of 12.
1 x 12
2 x 6
3 x 4
6 x 2
12 x 1

In the following factor pairs we still have an area (product) of 12 but our factors have changed. The factors on the left double every time, and the factors on the right halve each time. This strategy can work with several different problems but some problems do not lend themselves to doubling and halving.

This strategy helps because it changes the problem into simpler numbers. Once you reach a point where the numbers are easy for you to work with, then you stop doubling/halving and solve the problem.

\[
\begin{align*}
12 \times 25 & \quad 35 \times 8 \\
\div 2 & \quad \times 2 \\
6 \times 50 & \quad 70 \times 4 \\
\div 2 & \quad \times 2 \\
3 \times 100 = 300 & \quad 140 \times 2 = 280
\end{align*}
\]

Here are some problems you can use that support "Doubling and Halving."

<table>
<thead>
<tr>
<th>8 x 125</th>
<th>345 x 8</th>
<th>26 x 12</th>
<th>4 x 140</th>
<th>16 x 35</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 x 180</td>
<td>36 x 14</td>
<td>12 x 15</td>
<td>32 x 90</td>
<td>180 x 6</td>
</tr>
</tbody>
</table>
“Breaking Factors Into Small Factors”

Strategy

The associative property is the heart of this strategy. It is a great mental strategy to use when the problems become larger. One of the factors then can be changed to a one-digit multiplier.

\[
\begin{align*}
12 \times 25 & = (4 \times 25) + (4 \times 25) + (4 \times 25) \\
100 + 100 + 100 & = 300 \\
\end{align*}
\]

Students are comfortable with money amounts so they might approach this problem by breaking the 12 into 3 groups of 4.

\[
\begin{align*}
(4 \times 25) + (4 \times 25) + (4 \times 25) & = 3 \times (4 \times 25) \\
12 \times 25 & = 3 \times (4 \times 25) \\
\end{align*}
\]

Help them connect their thinking to the associative property by recording the problem as \(3 \times (4 \times 25)\). Have a discussion about whether \(12 \times 25\) is the same as \(3 \times 4 \times 25\). This will lead you into a discussion about associative property.

\[
\begin{align*}
12 \times 25 & = (12 \times 5) \times 5 \\
60 \times 5 & = 300 \\
\end{align*}
\]

We can also use the associative property and knowledge about factorization to think of 25 as \(5 \times 5\).

The array can help students understand how the associative property works with \(12 \times 25\). This array shows how the problem can be shown as 5 groups of \(12 \times 5\).

\[
\begin{array}{ccccc}
25 \\
\hline
5 & 5 & 5 & 5 & 5 \\
\hline
12 & 12 & 12 & 12 & 12 \\
\hline
\times 5 & \times 5 & \times 5 & \times 5 & \times 5 \\
\end{array}
\]
Here are some problems you can use that support “Breaking Factors Into Smaller Factors.”

<table>
<thead>
<tr>
<th>12 x 25</th>
<th>8 x 35</th>
<th>16 x 25</th>
<th>24 x 15</th>
<th>72 x 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>24 x 9</td>
<td>16 x 45</td>
<td>32 x 8</td>
<td>12 x 15</td>
<td>36 x 15</td>
</tr>
</tbody>
</table>