Unit Five Information

Curriculum Map: Comparing and Contrasting Functions

Content Descriptors:
Concept 1: Construct and Compare Linear, Quadratic & Exponential Models and Solve Problems
Concept 2: Interpret expressions for functions in terms of the situation they model and build new functions from existing functions
Concept 3: Understand the concept of a functions and use function notation
Concept 4: Interpret functions that arise in applications in terms of the context and analyze functions using different representations

Content from Frameworks: Comparing and Contrasting Functions

Georgia Milestones Study Guide

Unit Length: Approximately 20 days
**Unit Rational:**
In this unit, students deepen their understanding of linear, quadratic, and exponential functions as they compare and contrast the three types of functions. Students distinguish between additive and multiplicative change and interpret arithmetic sequences as linear functions and geometric sequences as exponential functions. Students compare characteristics of linear, quadratic, and exponential functions. Students observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Students select from among these functions to model phenomena.

### Prerequisites: As identified by the GSE Frameworks
- Understand and be able to explain what a function is.
- Determine if a table, graph or set of ordered pairs is a function.
- Distinguish between linear and non-linear functions.
- Write linear and exponential equations and use them to model real-world situations.
- Understand and interpret key features of graphs.
- Solve linear equations, inequalities, and systems of equations.
- Graph the solution set to a linear inequality in two variables.
- Perform addition, subtraction, and multiplication of polynomials.
- Simplify radical expressions.
- Factor quadratic expressions.
- Solve quadratic equations in one variable.

**Length of Unit:** 20 Days

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<thead>
<tr>
<th>Concept 1</th>
<th>Concept 2</th>
<th>Concept 3</th>
<th>Concept 4</th>
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<tbody>
<tr>
<td>Construct and Compare Functions</td>
<td>Interpret expressions in context</td>
<td>Graphing polynomial functions.</td>
<td>Application of Polynomial functions and finding the inverse</td>
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<table>
<thead>
<tr>
<th>GSE Standards</th>
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TCSS 7/20/2016
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<tbody>
<tr>
<td>Distinguish between situations that can be modeled with linear functions and with exponential functions.</td>
<td>Interpret the parameters in a linear (f(x) = mx + b) and exponential (f(x)=a•d^x) function in terms of context. (In the functions above, “m” and “b” are the parameters of the linear function, and “a” and “d” are the parameters of the exponential function.) In context, students should describe what these parameters mean in terms of change and starting value.</td>
<td>Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. (This can be shown by algebraic proof, with a table showing differences, or by calculating average rates of change over equal intervals).</td>
<td>Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.</td>
<td>Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.</td>
<td>Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.</td>
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<td>Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.</td>
<td>Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.</td>
<td>Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).</td>
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<td>Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If f is a function, x is the input (an element of the domain), and f(x) is the output (an element of the range). Graphically, the graph is y = f(x).</td>
<td>MGSE9-12.F.IF.4</td>
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<td>Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.</td>
<td>Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</td>
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| MGSE9-12.F.IF.6 | MGSE9-12.F.IF.7 | MGSE9-12.F.IF.8 | MGSE9-12.F.IF.9 | Compare properties of two functions each represented in a different way (algebraically,
graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.

**MGSE9-12.F.BF.3**
Identify the effect on the graph of replacing \( f(x) \) by \( f(x) + k, k f(x), f(kx), \) and \( f(x + k) \) for specific values of \( k \) (both positive and negative); find the value of \( k \) given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

<table>
<thead>
<tr>
<th>Lesson Essential Question</th>
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<tr>
<td>✓ How is a relation determined to be linear, quadratic or exponential? ✓ What types of functions do I use to represent and solve real-world problems? ✓ How can we use real world situations to construct and compare linear, quadratic and exponential models? ✓ How do I use graphs to represent and solve real-world equations? ✓ How do I use different representations to analyze linear and exponential functions? ✓ How do I build a linear or exponential function that models a relationship between two quantities? ✓ How do I build new functions from existing functions?</td>
<td>✓ How do I identify the parts of equations? ✓ How do I interpret expressions for functions in terms of the situation they model? ✓ How do I find the parameters of a function and what does it represent in terms of context?</td>
<td>✓ Why is the concept of a function important and how do I use function notation to show the situations they model?</td>
<td>✓ How do I interpret functions that arise in applications in terms of context? ✓ What are the specific features that distinguish the graphs of linear, quadratic and exponential functions from one another?</td>
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<table>
<thead>
<tr>
<th>Vocabulary</th>
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<tr>
<td>Arithmetic Sequence</td>
<td>Coefficient</td>
<td>Domain</td>
<td>Continuous</td>
</tr>
<tr>
<td>Average Rate of Change</td>
<td>Expression</td>
<td>Exponential Model</td>
<td>Discrete</td>
</tr>
<tr>
<td>Constant Rate of Change</td>
<td>Factor</td>
<td>Linear Model</td>
<td>End Behaviors</td>
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<tr>
<td>Exponential Function</td>
<td>Horizontal shift</td>
<td>Range</td>
<td>Root</td>
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<tr>
<td>Interval Notation</td>
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<td>X-intercept</td>
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## TCSS – GSE Algebra 1– Unit 5

<table>
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<tr>
<th>Even Function</th>
<th>Explicit Expression</th>
<th>Geometric Sequence</th>
<th>Linear Function</th>
<th>Odd Function</th>
<th>Quadratic equation</th>
<th>Quadratic function</th>
<th>Recursive Formula</th>
<th>Slope</th>
<th>Parameter</th>
<th>Term</th>
<th>Vertical Translation</th>
<th>Y-intercept</th>
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### Resources – Concept 1
- Even/Odd Function Notes
- Even/Odd Function Practice
- Comparing Functions Practice
- Building and graphing functions – linear vs exponential task
- Comparing Sequences Task

### Concept 1
**Differentiated Activities**
- These tasks were taken from the GSE Frameworks.
  - FAL.Birthday Gifts and Turtle Problem
  - FAL.Exploring Paths

### Resources – Concept 2
- Comparing Functions parameters

### Concept 2
**Differentiated Activities**

### Resources – Concept 3
- Comparing Functions Notes
- Comparing Linear, Quadratic, and Exponential Models Graphically

### Concept 3
**Differentiated Activities**

### Resources – Concept 4
- Graphic Organizer
- Compare/Contrast Functions
- Unit Review  KEY

### Concept 4
**Differentiated Activities**
- FAL Comparing Investments

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*These tasks were taken from the GSE Frameworks.*
TCSS – GSE Algebra 1– Unit 5

At the end of Unit student’s should be able to say “I can…”

- Deepen their understanding of linear, quadratic, and exponential functions as they compare and contrast the three types of functions.
- Understand the parameters of each type of function in contextual situations.
- Interpret linear, quadratic, and exponential functions that arise in applications in terms of the context.
- Analyze linear, quadratic, and exponential functions and model how different representations may be used based on the situation presented.
- Construct and compare characteristics of linear, quadratic, and exponential models and solve problems.
- Recognize that exponential and quadratic functions have a variable rate of change while linear functions have a constant rate of change.
- Distinguish between additive and multiplicative change and construct and interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.
- Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
- Define and use function notation, evaluate functions at any point in the domain, give general statements about how \( f(x) \) behaves at different regions in the domain (as \( x \) gets very large or very negative, close to 0 etc.), and interpret statements that use function notation.
- Explain the difference and relationship between domain and range and find the domain and range of a linear, quadratic, or exponential function from a function equation, table, or graph.
- Interpret \( x \) and \( y \) intercepts, where the function is increasing or decreasing, where it is positive or negative, its end behaviors, given the graph, table or algebraic representation of a linear, quadratic, or exponential function in terms of the context of the function.
- Find and/or interpret appropriate domains and ranges for authentic linear, quadratic, or exponential functions.
- Calculate and interpret the average rate of change over a given interval of a function from a function equation, graph or table, and explain what that means in terms of the context of the function.
- Explain the relationship between the domain of a function and its graph in general and/or to the context of the function.
- Accurately graph a linear function by hand by identifying key features of the function such as the \( x \)- and \( y \)-intercepts and slope.
- Discuss and compare different functions (linear, quadratic, and/or exponential) represented in different ways (tables, graphs or equations). Discussion and comparisons should include: identifying differences in rates of change, intercepts, and/or where each function is greater or less than the other.
- Write a function that describes a linear, quadratic, or exponential relationship between two quantities.
- Construct and compare linear, quadratic, and exponential models and solve problems.