## Unit 2: Reasoning with Linear Equations and Inequalities

### A.CED.1
Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear functions.

- The perimeter of a rectangle is 54 cm. If the length is 2 cm more than a number, and the width is 5 cm less than twice the same number, what is the number?
- There are 156 laptops and desktop computers in a lab. There are 8 more laptops than desktop computers. What is the total number of laptops in the lab?

### A.CED.2
Create linear equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase “in two or more variables” refers to formulas like the compound interest formula, in which \( A = P(1 + \frac{r}{n})^n \) has multiple variables.)

- The following graph shows the revenue (or income) a company makes from designer coffee mugs and the total cost (including overhead, maintenance of machines, etc.) that the company spends to make the coffee mugs.

  a. What is the meaning of the point (0, 4000) on the total cost line?
  
  b. What are the coordinates of the intersection point? What is the meaning of this point in this situation?
  
  c. Create linear equations for revenue and total cost in terms of units produced and sold. Verify the coordinates of the intersection point.
  
  d. Profit for selling 1,000 units is equal to revenue generated by selling 1,000 units minus the total cost of making 1,000 units. What is the company’s profit if 1,000 units are produced and sold?

### A.CED.3
Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

- A company produces packs of pencils and pens.
  - The company produces at least 100 packs of pens each day, but no more than 240.
  - The company produces at least 70 packs of pencils each day, but no more than 170.
  - A total of less than 300 packs of pens and pencils are produced each day.
  - Each pack of pens makes a profit of $1.25.
  - Each pack of pencils makes a profit of $0.75.

  What is the maximum profit the company can make each day?

  A) $338.75  
  B) $344.25  
  C) $352.50  
  D) $427.50

### A.CED.4
Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. Examples: Rearrange Ohm’s law \( V = IR \) to highlight resistance \( R \)

- The density of an object can be found using the formula \( D = \frac{m}{v} \)
  - \( m \) is the mass of the object
  - \( v \) is the volume of the object

  Which formula could be used to determine the volume of the object?

  A) \( v = mD \)  
  B) \( v = \frac{m}{D} \)  
  C) \( v = \frac{m}{P} \)  

### A.REI.1
Using algebraic properties and the properties of real numbers, justify the steps of a simple, one-solution equation. Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties.

- The steps in Raya’s solution to \( 2.5(6.25x + 0.5) = 11 \) are shown.

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5(6.25x + 0.5) = 11</td>
<td>Given</td>
</tr>
<tr>
<td>15.625x + 1.25 = 11</td>
<td>Multiplication Property of Equality</td>
</tr>
<tr>
<td>15.625x = 9.75</td>
<td>Subtraction Property of Equality</td>
</tr>
<tr>
<td>x = 0.624</td>
<td>?</td>
</tr>
</tbody>
</table>

Select the correct reason for line 4 of Raya’s solution.

A) Closure property  
B) Distributive property  
C) Addition property of equality  
D) Division property of equality
### A.REI.3
Solve linear equations and inequalities in one variable including equations with coefficients represented by letters. For example, given $ax + 3 = 7$, solve for $x$.

What is the value of $n$ in the equation $3n - 8 = 32 - n$?

- A) $-10$
- B) $-6$
- C) $6$
- D) $10$

What is the value of $x$ in the equation $13x - 2(x + 4) = 8x + 1$?

- A) $1$
- B) $2$
- C) $3$
- D) $4$

### A.REI.5
Show and explain why the elimination method works to solve a system of two-variable equations.

Solve this system of equations by elimination.

\[
\begin{align*}
3x - 2y &= 7 \\
2x - 3y &= 3
\end{align*}
\]

Explain which variable you eliminated first and why.

### A.REI.6
Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

What is the solution to this system of equations?

\[
\begin{align*}
y &= -3x - 2 \\
6x + 2y &= -4
\end{align*}
\]

- A) $(6, 2)$
- B) $(1, -5)$
- C) no solution
- D) infinitely many solutions

The math club sells candy bars and drinks during football games.

- 60 candy bars and 110 drinks will sell for $265.
- 120 candy bars and 90 drinks will sell for $270.

How much does each candy bar sell for?

A candy company sells cases of chocolate bars. The company has fixed costs of $30,000, and each case of chocolate bars costs an additional $5 to make. The company sells each case for $10. The graph of a system of linear equations representing this company's costs and revenue for manufacturing and selling $x$ cases of chocolate bars is shown below.

How many cases of chocolate bars will this company need to sell in order for costs and revenue to be equal?

- A) 3,500
- B) 6,000
- C) 35,000
- D) 60,000

### A.REI.10
Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.

You and a friend want to go to a local restaurant for dinner and you have a $25 gift card. List some possible combinations of prices for your meal and your friend’s meal so that you spend the exact amount on the gift card.

### A.REI.11
Using graphs, tables, or successive approximations, show that the solution to the equation $f(x) = g(x)$ is the $x$-value where the $y$-values of $f(x)$ and $g(x)$ are the same.

Graph the equations $y = 17x + 1$ and $y = -24x + 68$

a. Find the point of intersection.

b. Explain why the solution to the equation $17x + 1 = -24x + 68$ will give the same $x$-value as the point of intersection in part a.
### A.REI.12
Graph the solution set to a linear inequality in two variables.

Which graph represents the inequality $-2x + 3y > 12$?

- [Image of graphs A, B, C, D]

### F.BF.1
Write a function that describes a relationship between two quantities.

Georgia is purchasing treats for her classmates. Georgia can spend exactly $10.00 to purchase 25 fruit bars, each equal in price. Georgia can also spend exactly $10.00 to purchase 40 granola bars, each equal in price.

Write an equation that can be used to find all combinations of fruit bars ($x$) and granola bars ($y$) that will cost exactly $10.00.

### F.BF.1a
Determine an explicit expression and the recursive process (steps for calculation) from context. For example, if Jimmy starts out with $15 and earns $2 a day, the explicit expression "2x+15" can be described recursively (either in writing or verbally) as "to find out how much money Jimmy will have tomorrow, you add $2 to his total today."

The sequence below shows the total number of days Francisco had used his gym membership at the end of weeks 1, 2, 3, and 4.

4, 9, 14, 19, ...

Assuming the pattern continued, which function could be used to find the total number of days Francisco had used his gym membership at the end of week $n$?

A) $f(n) = n + 5$
B) $f(n) = 5n - 1$
C) $f(n) = 5n + 4$
D) $f(n) = n^2$

### F.BF.2
Write arithmetic and geometric sequences recursively and explicitly, use them to model situations, and translate between the two forms. Connect arithmetic sequences to linear functions.

The table below shows the cost of a pizza based on the number of toppings.

<table>
<thead>
<tr>
<th>Number of Toppings ($n$)</th>
<th>Cost ($C$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$12</td>
</tr>
<tr>
<td>2</td>
<td>$13.50</td>
</tr>
<tr>
<td>3</td>
<td>$15</td>
</tr>
<tr>
<td>4</td>
<td>$16.50</td>
</tr>
</tbody>
</table>

Which function represents the cost of a pizza with $n$ toppings?

A) $C(n) = 12 + 1.5(n - 1)$
B) $C(n) = 1.5n + 12$
C) $C(n) = 12 + n$
D) $C(n) = 12n$
Consider a sequence given by the formula \( a_n = a_{n-1} - 5 \), where \( a_1 = 12 \) and \( n \geq 2 \).

a. List the first five terms of the sequence.

b. Write an explicit formula.

Which representation does not show \( y \) as a function of \( x \)?

A) 

B) 

C) \( \{(-1, -2), (0, 1), (2, 4), (7, 7)\} \)

D) 

Find \( f(-1) \) when \( f(x) = -5x + 3 \)

A car rental company charges a flat rate of $20 plus $0.22 per mile driven. The function \( f(m) = 20 + 0.22m \) represents this situation.

Explain what \( f(150) \) means in this context.

Look at the sequence in this table.

<table>
<thead>
<tr>
<th>( n )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_n )</td>
<td>-1</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
</tr>
</tbody>
</table>

Which function represents the sequence?

A) \( a_n = a_{n-1} + 1 \)

B) \( a_n = a_{n-1} + 2 \)

C) \( a_n = 2a_{n-1} - 1 \)

D) \( a_n = 2a_{n-1} - 3 \)

The graph shows the average speed of a runner during a marathon.

Explain what the slope of this graph represents.
For a science project, Maurice and Darlene raced rubber-band cars and recorded their results. They both used $x$ to represent the time in seconds a car has traveled and $y$ to represent the distance in meters a car is from a given point.

Maurice recorded his results in the table shown.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>13</td>
</tr>
</tbody>
</table>

Darlene recorded her results in the graph shown.

Part A
At the end of 2 seconds, which car is the greatest distance from the given point? Explain your answer.

Part B
Which car is traveling at the fastest speed? Show your work or explain your answer.

Part C
At what time will both cars be the same distance from the given point? Show your work or explain your answer.

F.IF.5
Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function.

A plane can carry a maximum cargo weight of 160,000 pounds. A company uses one of these planes to ship 2,000-pound containers. The total cargo weight is a function of the number of containers in the plane. What is the greatest value in the domain for this situation?

F.IF.6
Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

The table below shows the distance a car has traveled.

<table>
<thead>
<tr>
<th>Minutes</th>
<th>Distance Traveled (in miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20</td>
</tr>
<tr>
<td>25</td>
<td>40</td>
</tr>
<tr>
<td>50</td>
<td>60</td>
</tr>
<tr>
<td>75</td>
<td>80</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

What is the meaning of the slope of the linear model for the data?

A) The car travels 5 miles every minute.
B) The car travels 4 miles every minute.
C) The car travels 4 miles every 5 minutes.
D) The car travels 5 miles every 4 minutes.

The graph shows the number of sales for a musical CD the first 10 weeks it was sold in stores.

Which of the following statements is best supported by the graph?

A) The number of sales decreased by one hundred thousand every 3 weeks
B) The number of sales increased by one hundred thousand every 3 weeks
C) The number of sales decreased by one hundred thousand every 4 weeks
D) The number of sales decreased by one hundred every 3 weeks
Given the graph of \( y = 2x + 2 \) below, find the x- and y-intercepts.

Which equation represents the line shown in the graph below?

A) \( y = \frac{2}{3}x + 4 \)
B) \( y = \frac{2}{3}x - 6 \)
C) \( y = \frac{3}{2}x + 4 \)
D) \( y = \frac{3}{2}x - 6 \)

The figure shows the graph of \( f(x) = 1 \cdot 2x + c \).

a. Find the value of \( c \).

b. If the graph of \( f \) intersects the y-axis at \( B \), find the coordinates of \( B \).

c. Find the area of triangle \( AOB \)
F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.

A phone call using a prepaid card consists of a fixed fee to place the call plus an additional fee for each minute of the call. The cost of an $n$-minute phone call with a card from Company A is $A(n) = 0.99 + 0.25n$, where $n$ is a positive integer.

The cost of an $n$-minute phone call with a card from Company B is shown in the graph below.

Which statement below must be true?

A) The per minute fee for Company B is greater than Company A.
B) The fixed fee for Company B is greater than Company A.
C) A call using Company B will always cost more than the same length call using Company A.
D) A two-minute call with Company B is less than Company A.